

IN-SITU TESTING WITH Diffeomorphic Mapping

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Summary: We present a novel method in computing the displacement vector fields induced by mechanical testing of material. Our approach generates a local description of motion and can therefore be used to describe local behavior of material.

1 INTRODUCTION

Digital Volume Correlation (DVC) is the method of choice for the estimation of displacements originating from in-situ testing in volumetric computed tomography. Even for images of large dimension, DVC produces reliable results of displacement fields that can be used for strain calculation of load or tensile testing. Yet DVC shows by construction a deficit that excludes its usage in the displacement estimation of spatial thin structures such as foams: It must be understood as an average deformation in the interrogated subvolumes [1]. In addition, it produces displacement fields that are rather continuous, in the sense that small cracks in the interrogated subvolumes are smoothed out and are not taken into account when calculating strain maximas. Even the combination with local strategies as BSpline-Registration and refinement of strong deforming areas fails in applications, where highly local behavior is of interest, see for example [2]: For ceramical foams it is known that collective failure of whole levels is preceded by the rupture of single struts. This discontinuous deformation usually only extends to a small number of voxels, meaning that the search window and node spacing of DVC must be chosen very fine, resulting in poor runtime and memory usage.

We therefore propose to compute displacements based on Large Deformation Diffeomorphic Metric Mapping (LDDMM), a method originating from Medical Image Registration. In contrast to DVC, the algorithm produces a smooth mapping for each image position. That is, we do not solve a minimization for subvolumes but for all image voxels and remain with information per voxel, making it possible to evaluate displacements even for small struts of only a few voxels diameter.

2 LARGE DEFORMATION Diffeomorphic METRIC MAPPING

Let $\Omega \subset \mathbb{R}^3$ be a bounded domain. DVC seeks to find a suitable mapping $y: \Omega \subset \mathbb{R}^3 \rightarrow \mathbb{R}^3$, $x \mapsto x + v(x)$ that transforms a template image $\mathcal{T}: \Omega \subset \mathbb{R}^3 \rightarrow \mathbb{R}$ to a reference image $\mathcal{R}: \Omega \subset \mathbb{R}^3 \rightarrow \mathbb{R}$ by linear displacements v . The transformed image, here denoted by u , should be minimal with respect to some distance measure. Possible choices for these measures are sum-of-squared differences or normalized cross correlation. We therefore minimize a functional of the form

$$\min_{v,u} \{ \mathcal{J}(v, u) := \mathcal{D}(u, \mathcal{R}) + \alpha \mathcal{S}(v) \}, \quad (1)$$

where \mathcal{D} is the distance measure, and \mathcal{S} is a regularization functional to overcome ill-posedness of the minimization problem and often involve image intensity gradients, see for example [3]. In contrast to DVC, we now want to consider y as a diffeomorphism: Instead of calculating displacements of subvolumes, we calculate a path along which an image position with a corresponding grey value evolves. That is, we reconstruct now u as a

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time series $u(x, t)$ that interpolates smoothly between $\mathcal{T}(x) = u(x, 0)$ and $\mathcal{R} = u(x, 1)$. The time-series can be reconstructed by solving a partial differential equation. Assuming mass conservation of the examined material, we solve (1) constrained by

$$\mathcal{C}(u, v) = \begin{cases} \partial_t u(x, t) + v(x, t) \cdot \nabla u(x, t) = 0 \\ u(x, 0) = \mathcal{T}(x), \end{cases} \quad (2)$$

where v now is a velocity vector field. We use Lagrangian methods to eliminate the PDE-constraint, in detail, the PDE (2) is reconstructed along its characteristics. The resulting unconstrained minimization problem can then be solved by a memory-distributed gradient-descent scheme to make the method applicable to large 3D images. After optimization, we remain with a vector of new image positions. Subtracting them from the original positions gives us displacement that can be used for further analysis of the mechanical behavior.

3 EXPERIMENTAL METHOD AND RESULTS

We use the implementation of Mang et al. [4] for LDDMM, as it offers support for high-resolution 3D images by iterative optimization methods and compare it to the DVC implementation of Tudisco et al. [5]. We test both algorithms with a cylindrical probe of polyurethane adhesive with lead balls to enhance contrast. Tensile tests were performed at Fraunhofer IMWS. Tension is applied via the circular cross section of the cylinder, however, the main impact is within the rectangular cross sections which bend to the inside as in Figure 1a. The displacement fields in Figures 1b and 1c have the circular cross section as viewing plane, and one can clearly see that our proposed algorithm describes the desired displacement fields. Most prominent is the movement with pulling direction from left to right but in addition circular displacements for the necking of the material are computed, whereas DVC is only showing displacements in tensile direction. Therefore we consider it as a promising tool for the estimation of displacement fields and strain estimation even for structures of thin spatial information.

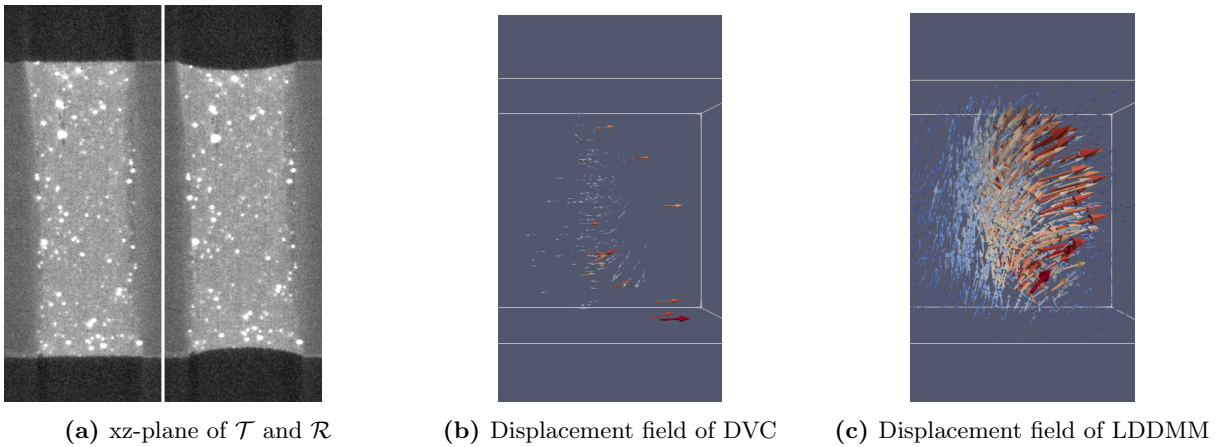


Figure 1: Results of tensile testing of polyurethane adhesive with lead balls. The size of the dataset was $502 \times 506 \times 1000$ pixels with a total probe size of $8.02 \times 8.14 \times 16.1$ mm, and tension was applied in z-direction (from left to right). For visualization, the results were cropped to the region of interest and vector fields were augmented.

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